

中國古算今譚
— 從傳統數學
至西學輸入
至現代課堂數學III：
現代中學生 / 清代康熙帝
初遇上代數方程

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中小學數學課堂上傳授的基本知識和技能，很大部分已經有數百年以至數千年的歷史。從古代至十七世紀的東西方數學典籍當中，記載了相當多的部份。回顧從中國古代至十六世紀的傳統數學，至明末清初西學東漸合流會通，演變成為二十世紀以降在華人教育圈中的現代中小學數學內容。這方面的探討，不只有其數學意義，也富有文化意義，對教與學，都有裨益。

前年五月的講座（「中國古算今譚 — 從傳統數學至西學輸入至現代課堂數學」）可視為這項嘗試的「前傳」，去年五月的講座討論中學的幾何課程，今年五月的講座將會集中討論中學代數課程中有關解方程的部份。

當年大家初次學習解代數方程，設立未知數獲得答案，有沒有感到這個方法很奇妙呢？抑或覺得只是按程式逐步演算，有些枯燥乏味呢？

中國數學家自秦漢以降，便懂得解今天看來是一次或二次方程的問題；唐宋以後，更能解高次方程。

但那些都不全是今天中學生在課堂上學到的內容，今天中學生在課堂上學到的內容，是十七世紀在西歐發展起來解方程的方法，也是在十八世紀初法國耶穌會傳教士講授予康熙帝的代數新法。看看當年康熙帝的學習過程，說不定會有某方面的啟發，能夠為今天中學代數的教與學帶來一點啟示。

<p>秦漢或更早 (公元前三世紀至公元一世紀)</p>	<p>《九章算術》</p>	<p>開方術, 帶從開方術 (聯立線性)方程(組)</p>
<p>晉 (三世紀)</p>	<p>《孫子算經》</p>	<p>(聯立線性)同餘方程(組)</p>
<p>南北朝 (五世紀)</p>	<p>《張丘建算經》</p>	<p>不定方程</p>
<p>唐 (七世紀)</p>	<p>王孝通《輯古算經》</p>	<p>三次方程</p>
<p>宋、元 (九世紀至十四世紀)</p>	<p>《楊輝算法》: 劉益 《議古根源》、賈憲 《黃帝九章算法細草》 秦九韶《數書九章》 李冶《測圓海鏡》、 《益古演段》 朱世傑《算學啟蒙》 、《四元玉鑑》</p>	<p>釋鎖、演段、 增乘開方法 正負開方術、大衍求一術 天元術</p>
<p>明清之際 (十七世紀初)</p>	<p>利瑪竇、李之藻 《同文算指》</p>	<p>筆算</p>
<p>清 (十七、十八、十九世紀)</p>	<p>梅文鼎《少廣補遺》 《數理精蘊》 「談天三友」(焦循、 汪萊、李銳) 方中通、焦循、 駱騰鳳、丁取中、 時曰醇 偉烈亞力、李善蘭 傅蘭雅、華蘅芳</p>	<p>借根方法 方程論 不定方程 譯《代數學》 譯《代數術》</p>

故事由十五、十六世紀西方的「探索年代」開始。當時歐洲人找到一條通到東方的海路，不同類別的人，因不同的理由來到東方，其中有
一批是傳教士。

傳教士除了宣傳福音外，還揭開了東西方兩大文明的知識和文化交流重要的
一頁。

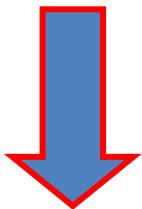
徐光啟和利瑪竇合作翻譯《原本》，掀起歐洲科學傳入中國的第一波浪潮，接著的

第二波（或如一些歷史學家形容為第一波的尾流）和第三波出現時已是清朝，這三波浪潮的出現，有相當不同的歷史背景。

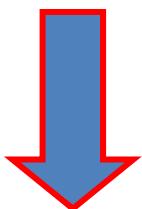
第一波取得的成就，看來很短暫，隨著明朝的衰亡消失。但現在回顧，我們還能看到它的長遠影響。

可是，當時打開了通往外間奇妙世界的小窗，很快便關上，一直要到二百年後，西方的堅船利炮強行打開一道大門，使這個古老大國在隨後的一百年受盡掠奪與羞辱，因而急切要知道西方世界的事物並狂熱學習。

十六世紀後期至
十七世紀中葉
(明朝期間)



十八世紀初
(清朝期間)



十九世紀晚期
(清朝期間)

「欲求超勝，
必須會通。」

**(In order to surpass
we must try to
understand and to
synthesize.)**

「西學中源」

**(Western learning
has its origin in
Chinese learning.)**

「師夷長技以制夷」

**(Learn the strong
techniques of the
“[Western] barbarians”
in order to control
them.)**

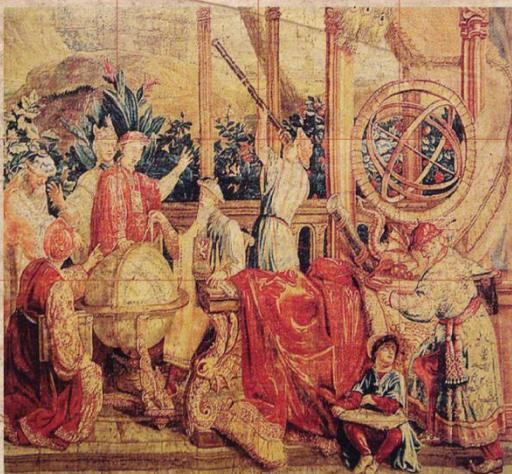
Faculty of Science
Public Lecture Series



THE EMPEROR'S NEW LESSONS

- HOW AND WHY DID -

EMPEROR KANGXI LEARN WESTERN SCIENCE AND MATHEMATICS?



by Professor
Siu Man Keung



Department of Mathematics
The University of Hong Kong

ABSTRACT

Since 1688 there went on inside the Imperial Court lessons in mathematics and astronomy taught to Emperor Kangxi by the Jesuits, leading to the establishment of an Office of Mathematics at Mengyangzhai (Studio for the Cultivation of the Youth) in 1713 and the ten-year project in compiling the 100-volume *Luli Yuanjian* (Origins of Mathematical Harmonics and Astronomy). How successful did Emperor Kangxi learn the new lessons? What drove an emperor to learn Western science and mathematics in such earnest? This talk tries to examine this episode that would be better understood in a political context along with its educational context.

Date: Feb 17, 2012 (Fri)

Time: 5.30 pm
(light refreshments
from 5 pm)

Medium: English

Venue: Wang Gungwu Theatre,
Graduate House, HKU

Admission: Free

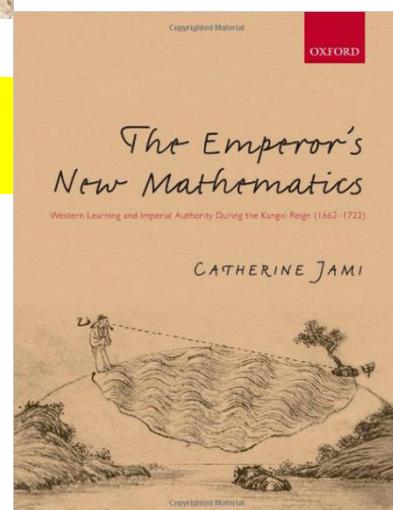
Please visit: <http://www.scifac.hku.hk/events/comm/2012/Kangxi> for seat reservation.
For enquiries, please call Miss Lee of Faculty of Science at 2241 5861.

M.K. Siu, **The Emperor's New Lessons : Why and How Did Emperor Kangxi Learn Western Science and Mathematics?**

Catherine Jami, *The Emperor's New Mathematics: Western Learning and Imperial Authority During the Kangxi Reign (1662-1722)*, Oxford University Press, 2011.

HKU
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Public
Lecture

2012.02.17



Emperor Kangxi (康熙) came to the throne before he turned seven in 1661 and reigned for more than sixty-one years until his death at the age of sixty-eight in 1722.



訓曰爾等惟知朕算術之精却不知我學算之故朕幼時欽天監漢官與西洋人不睦互相參劾幾至大辟楊光先湯若望於午門外九卿前當面賭測日影奈九卿中無一知其法者朕思已不知焉能斷人之是
非因自憤而學焉今凡入算之法累輯成書條分縷析後之學此者視此甚易誰知朕當日苦心研究之難也

《聖祖庭訓格言 (四庫全書)》

[Collection of Moral Instructions From Emperor Kangxi]



中法藝術文化的交會

康熙大帝與太陽王路易十四特展

展覽時間》2011.10/3 - 2012.1/3

展覽地點》國立故宮博物院 圖書文獻大樓一樓特展室

國立故宮博物院 國立歷史博物館 中國文物交流中心

**Emperor Kangxi and the Sun King Louis XIV,
Special Exhibition at Palace Museum, Taipei,
03-10-2011 to 03-01-2012.**



Emperor Kangxi (康熙)
(1654 – 1722)
reigned 1661 – 1722



King Louis XIV (路易十四)
Le Roi Soleil (Sun King)
(1638 – 1715)
reigned 1643 – 1715

“King’s Mathematicians” sent by Louis XIV to China in 1685 (reached Peking in 1688) — all French Jesuits

Jean de Fontaney (1643 – 1710) 洪若(翰)

Joachim Bouvet (1656 – 1730) 白晉

Jean-François Gerbillon (1654 – 1707) 張誠

Louis Le Comte (1655 – 1728) 李明

Claude de Visdelou (1656 – 1737) 劉應

[Guy Tachard (1651-1712) 塔夏爾 – stayed in Siam]

English translation of Bouvet's *Histoire de l'empereur de la Chine: présentée au roy (1697)* in 1699

“His Natural Genius is such as can be parallel'd but by few, being endow'd with a **Quick and piercing Wit, a vast Memory, and Great Understanding;** His constancy is never to be shaken by any sinister Event, which makes him the fittest Person in the World, not only to undertake, but also to accomplish Great Designs.”

“But, what may seem most surprising, is, that so Great a Monarch, who bears upon his shoulders the Weight of so vast an Empire, should apply himself with a great deal of Assiduity to, and have a true relish of all Sorts of useful Arts and Sciences.”



**Joachim Bouvet
(1656-1730)**

“During the space of two Years, Father Verbiest instructed him in the Usefulness of the best of the Mathematical Instruments, and in what else was most Curious in Geometry, the Statique, and Astronomy; for which purpose he wrote several Treatises.

He did the Honour to us four Jesuits, Missionaries then at Peking, to receive our Instructions, sometimes in the Chinese, sometimes in the Tartarian Language;

“Much about the same time,
Father **Anthony Thomas**, did
give him further Instruction
concerning the Use of the best
Mathematical Instruments, in
the Chinese language, and the
Practical part of Geometry and
Arithmatick, the principles of
which he had formerly been
taught by Father Verbiest. He
would also have us explain
him the Elements of Euclid in
the Tartarian Language, being
desirous to be well instructed
in them, as looking upon them
to be the Foundation, upon
which to build the rest.

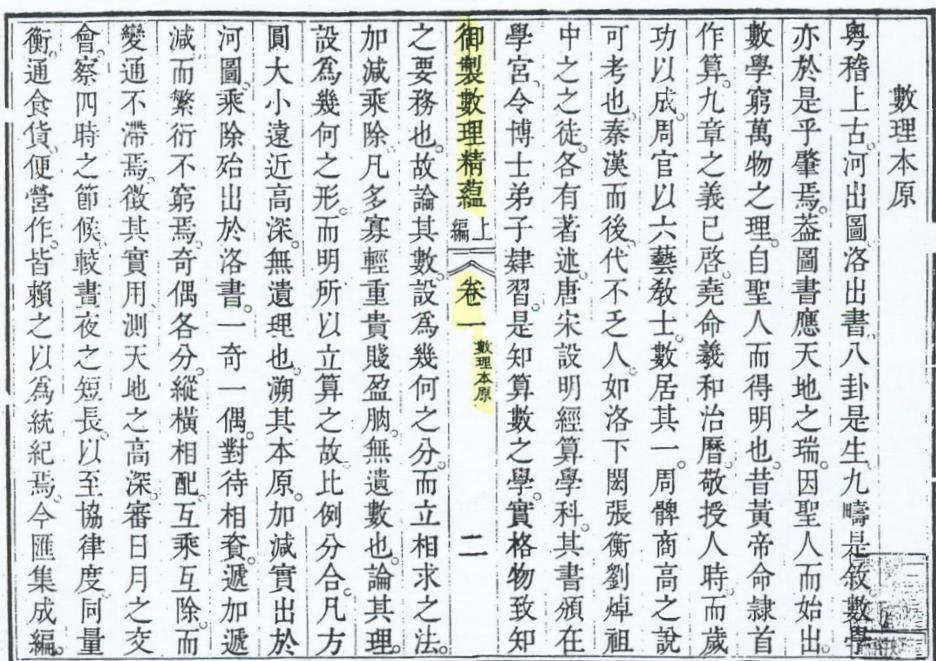
“He was so much delighted in the pursuit of these Sciences, that besides betwixt two and three Hours, which were set aside every day on purpose to be spent in our Company, he bestowed most of his leisure time, both in the day and at night in his Studies.”



Joachim Bouvet
(1656-1730)

Compilation of *Lü Li Yuan Yuan* (Origins of Mathematical Harmonics and Astronomy 律曆淵源) commissioned by Emperor Kangxi (project started in 1713, published in 1722/23)

- ❖ *Li Xiang Kao Cheng* (Compendium of Observational Computational Astronomy 曆象考成), 42 volumes.
- ❖ *Shu Li Jing Yun* (Collected Basic Principles of Mathematics 數理精蘊), 53 volumes.



- ❖ *Lü Lü Zheng Yi* (Exact Meaning of Pitchpipes 律呂正義), 5 volumes.

Ferdinand Verbiest (1623-1688) 南懷仁

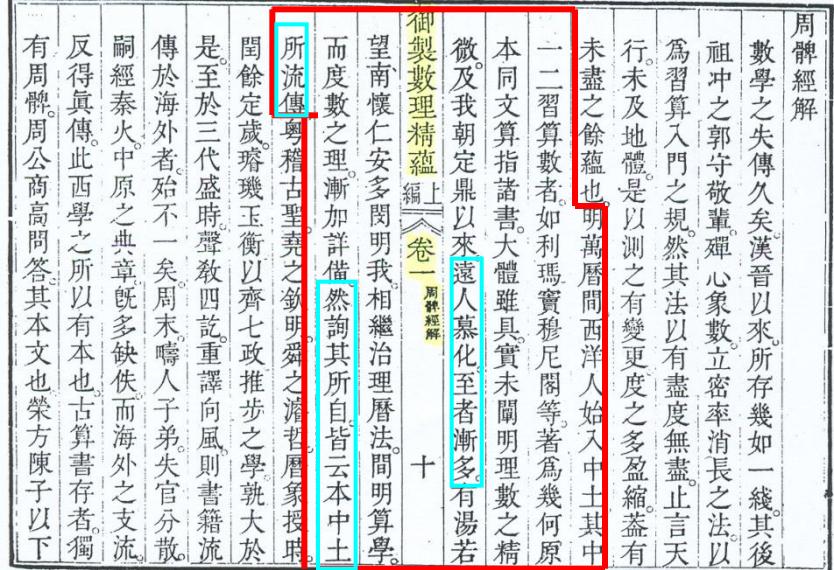
Antoine Thomas (1644-1709) 安多

Philippus – Maria Grimaldi (1639-1712) 閔明我

*Zhou Bi Jing Qie
(Explanation of the
classics Zhou Bi 周髀經
解) in Volume I of Shu Li
Jing Yun (Collected Basic
Principles of Mathematics
數理精蘊), 1722/23.*

陳厚耀、梅穀成、
何國宗、明安圖、
魏廷珍、王蘭生、
方苞、等人參與
編纂工作。

Johann Adam Schall von Bell
(1591-1666) 湯若望



Matteo Ricci (1552-1610) 利瑪竇
Jean-Nicolas Smogolenski (1611-
1656) 穆尼閣

Thomas Pereira (1645-1708) 徐日昇

Bernard-Kilian Stumpf (1655-1720) 紀理安

Jean-François Foucquet (1665-1741) 傅聖澤

Pierre Jartoux (1668-1720) 杜德美

Ignatius Kögler (1680-1746) 戴進賢

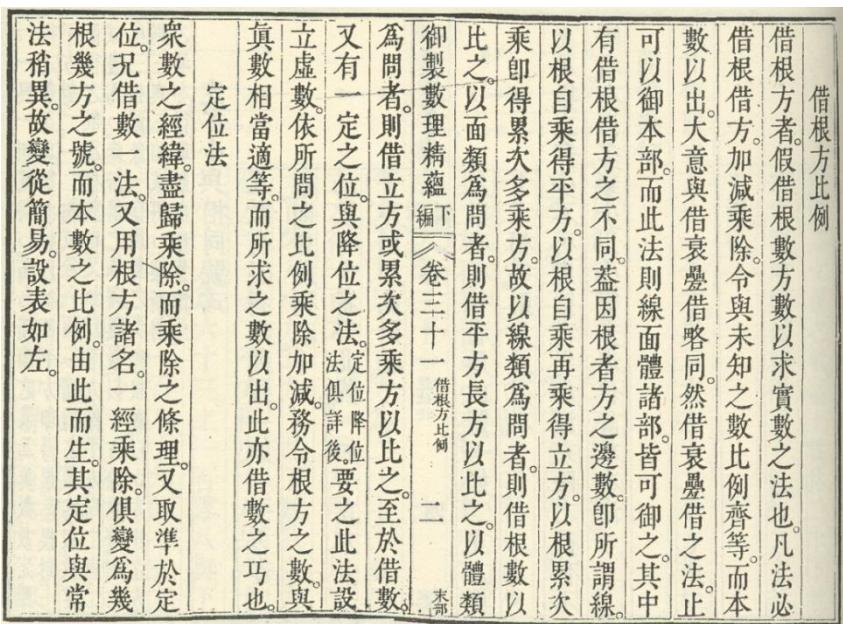
Antoine Gaubil (1689-1759) 宋君榮

André Pereira (1690-1743) 徐懋德

Michel Benoist (1715-1774) 蔣友仁

(in addition to “King’s Mathematicians”)

Jiegenfang Suanfa [借根方算法 method of borrowed root and powers]



François Viète, *De numerosa potestatum ad exegesim resolutione* (1600)



Antoine Thomas, *Synopsis Mathematica* (1685)



Antoine Thomas, 《算法纂要總綱》 [Outline of the Essential Calculations] and 《借根方算法》 [Method of Borrowed Root and Powers]
(between 1689-1695)



Book 31-36 of *Shuli Jingyun* [數理精蘊 Collected Basic Principles of Mathematics] (1712 - 1722/1723)



François Viète
(1540-1603)

*De numerosa potestatum
ad exegesim resolutione*
(1600)

Example: [For easier comprehension we adopt our accustomed mathematical language.]

Solve $x^2 + Ax = B$.

Let x_1 be a first approximation of a root.

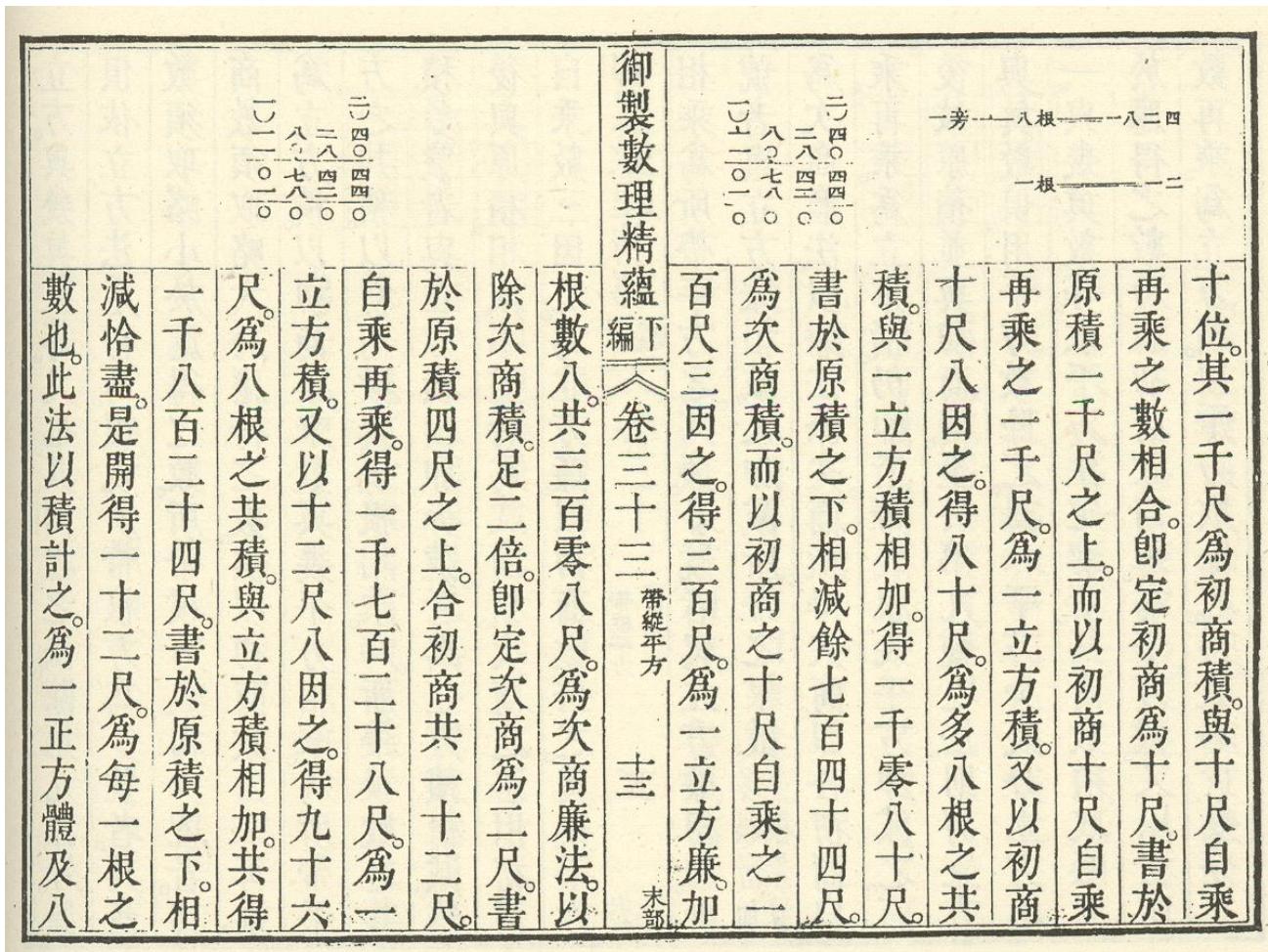
$$(x_1 + x_2)^2 + A(x_1 + x_2) = B.$$

$$x_1^2 + 2x_1x_2 + x_2^2 + Ax_1 + Ax_2 = B.$$

Neglecting x_2^2 , we obtain $x_2 = \frac{B - x_1^2 - Ax_1}{2x_1 + A}$.

$x_1 + x_2 = x_1 + \frac{B - x_1^2 - Ax_1}{2x_1 + A}$ is a better approximation. Iterate the process to get a better and better approximation of the root.

Book 33 of *Shuli Jingyun* 帶縱立方(extraction of cube root with accompanying number)



$$\begin{array}{r}
 1 \quad 2 \\
 \hline
 1 \quad 8 \quad 2 \quad 4 = 10^3 + 8 \times 10 \\
 1 \quad 0 \quad 8 \quad 0 \\
 \hline
 0 \quad 7 \quad 4 \quad 4 \\
 1 \quad 8 \quad 2 \quad 4 = 12^3 + 8 \times 12 \\
 \hline
 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

A problem in Book 33 of *Shuli Jingyun*

If the cube [of root] and eight roots are equal to 1824 *che*, how much is one root?

[In modern mathematical language, solve $x^3 + 8x = 1824$.]

Set $x_1 = 10$ as a first approximation.

$$(x_1 + x_2)^3 + 8(x_1 + x_2) = 1824$$

$$x_1^3 + 3x_1^2x_2 + 3x_1x_2^2 + x_2^3 + 8x_1 + 8x_2 = 1824$$

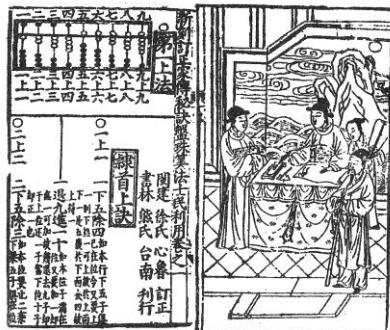
$$x_1^3 + 8x_1 = 1080$$

Neglecting x_2^2, x_2^3 , we obtain

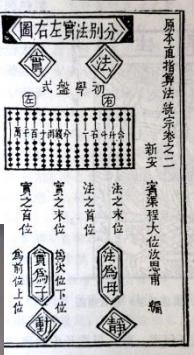
$$(3x_1^2 + 8)x_2 = 744, \text{ or } x_2 = \frac{744}{308} = 2.4\dots$$

Set $x_1 + x_2 = x_1 + 2 = 12$ as the next approximation. $12^3 + 8 \times 12 = 1824$, so $x = 12$ is a root.

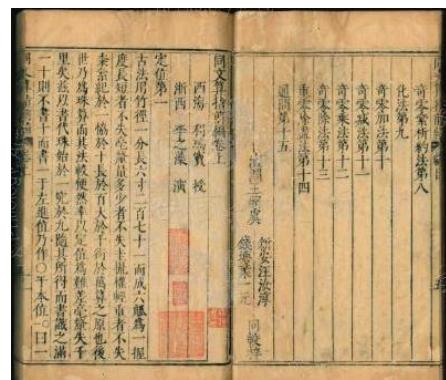
[If the answer is not exact, the process can be iterated to yield a better and better approximation to any decimal place.]



徐心魯
《盤珠算法》(1573)



程大位
《算法統宗》(1592)



自元中葉而後，由**籌算**演變而來的**珠算**日趨普遍，至十六世紀後期已漸取代籌算。明代有關珠算的主要著述，有徐心魯的《盤珠算法》(1573)與程大位的《算法統宗》(1592)。

十七世紀初，利瑪竇與李之藻合編的《同文算指》(1613)，把**筆算**引進中國，顯示了筆算較籌算及珠算均有其優越之處，特別是，**通過**記錄全部運算過程，方便理解數學運算的數理。

利瑪竇、李之藻，《同文算指》(1613)

M. K. Siu, *Tongwen Suanzhi* (同文算指) and transmission of bisuan (筆算 written calculation) in China: from an HPM (History and Pedagogy of Mathematics) viewpoint, *Journal for History of Mathematics*, 28(6) (2015), 311-320.

[...]後供奉內廷，蒙聖祖仁皇帝授以**借根方法**，且諭曰：西洋人名此書為**阿爾熱八達**，譯言**東來法也**。敬受而讀之、其法神妙，誠算法之指南。而竊疑**天元一之術**頗與相似，復取《授時曆草》觀之，乃渙如冰釋，殆名異而實同，非徒曰似之已也。

梅穀成 [MEI Juecheng 1681-1763]
《赤水遺珍 [*Pearls remaining in the red river*]》(1761)

夫元時學士著書，台官治曆，莫非此物，不知何故遂失其傳。猶幸遠人慕化，復得故物，東來之名，彼尙不能忘所自，而明人獨視為贅疣而欲棄之。噫！好學深思如唐顧二公，猶不能知其意，而淺見寡聞者又何足道哉！何足道哉！

天朝自居心態乎？

梅穀成 [MEI Juecheng 1681-1763]
《赤水遺珍 [*Pearls remaining in the red river*]》(1761)

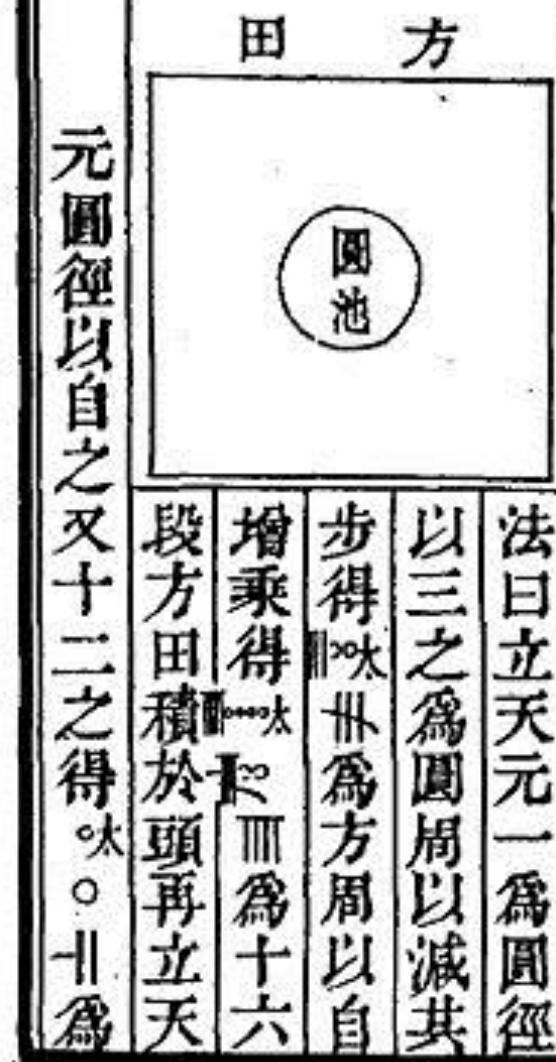
李冶 (1192-1279) 《益古演段》(1259)

第八問

百步間方圓周各多少

荅曰外方周二百四十步 內圓周六十

步



欽定四庫全書

益古演段

提要

臣等謹案益古演段三卷元李冶撰
據至元壬午硯壁序稱冶測圓海鏡

既已刻梓其親舊省掾李師徵復命

其弟師珪講治是編刊行是書在測

圓海鏡之後矣其曰益古演段者蓋
當時某氏算書_{益古演段序}但稱近世有
某是治已不知作者

益古演段 提要
一則不足識其真

益古演段自序

術數雖居六藝之末而施之人事則最爲切務

故古之博雅君子馬鄭之流未有不研精於此
者也其撰著成書者無慮百家然皆以九章爲
祖而劉徽李淳風又加注釋而此道益明今之

爲算者未必有劉李之工而褊心窮見不冒曉

然示人惟務隱互錯糅故爲溟涬黯黮惟恐學

者得窺其彷彿也不然則又以淺近徇俗無足

觀者致使軒轅隸首之術三五錯綜之妙盡墮

今有方田一段，內有圓池水占之，外
有地一十三畝七分半。只云內外方圓周
共相和得三百步，問方圓周各多少？
答曰：外方周二百四十步，內圓周六十步。
法曰：立天元一為圓徑，[....]

法曰:立天元一為圓徑 ,

以三之為圓周 , 以減共
步得 $\frac{1100}{-3x}$ 為方周 ,

以自增乘得 $\frac{\pi 0000}{-3x}$ 為十六
段方田積於頭 , 再立天
元圓徑以自之又十二之
得 $\frac{8}{-11}$ 為十六個圓池積 ,

以減頭位得 $\frac{\pi 0000}{-3x}$ 為
十六段如積。

寄左然後列真積一十三
畝七分半 , 以畝法通之
得三千三百步 , 又就分
母一十六通之得五萬
二千八百步。

與左相消得 $\frac{1100}{-3x}$,

開平方得二十步為圓池
徑 , 又三之為圓池周 。

Let x be the
diameter , then the
circumference is $3x$.
The perimeter is
 $300 - 3x$.

$$16 \times \text{area of square} \\ = 9x^2 - 1800x + 90000 .$$

$$16 \times \text{area of circle} \\ = 12x^2$$

$$16 \times \text{area of square} \\ \text{minus } 16 \times \text{area of} \\ \text{circle}$$

$$= -3x^2 - 1800x + 90000 .$$

$$16 \times 3300 = 52800 .$$

$$-3x^2 - 1800x + 90000 \\ = 52800 .$$

$$-3x^2 - 1800x + 37200 \\ = 0 .$$

$$x = 20 , \text{ and} \\ \text{circumference} = 60 .$$

الكتاب المختصر

في حساب الجبر و المقابلة

تصنيف

الشيخ الأجل أبي عبد الله محمد بن موسى

الخوارزمي

طبع في مدينة لندن

سنة ١٨٣٠ الميلادية

علي تسعه وثلاثين يتم المطح العظم الذي هو سطح رءة نبلغ ذلك كله اربعة وستين فاخذنا جذرها وهو ثمانية وهو اربع المطح العظم فإذا نقصنا منه مثل ما زدنا عليه وهو خمسة بقي ثلاثة وهو فلنج سطح آب الذي هو المال وهو جذرة والمالي تسعه وهذه صورته

	ج	
	ب	
٢٥		٥

واما مال واحد وعشرون درهما يعدل عشرة اجذاره فانا نجعل المال سطحا مربعا مجهول الاضلاع وهو سطح آب ثم نعم الـ سطحا متوازي الاضلاع عرضه مثل احد اضلاع سطح آب وهو فلنج دون والـ سطح آب فصار طول السطحين جميعا فلنج جهة وقد علمنا ان طره عشرة من العدد لأن كل سطح مربع مساوى الاصلان والزوايا فان احد اضلاعه مخروبا في واحد جذر ذلك المطح وفي اثنين جذراه فلما قال مال واحد وعشرون يعدل عشرة اجذاره علمنا ان طول فلنج آب جد عشرة اعداد لأن فلنج جد جذر المال نقصنا فلنج جهة بتصفيق على نقطه



Muhammed ibn Mūsā al-Khwārizmī (c.780–c.850) was a Persian mathematician and astronomer who lived in Baghdad during the first golden age of Islamic science, and whose name has given rise to the word *algorithm*. He wrote a book on the Hindu-Arabic positional method of counting and a treatise on algebra, both of which had an enormous influence in Europe when translated into Latin in the 12th century. The latter work, *Kitab al-jabr wa al-muqābalah* (the science of transposition and cancellation) was a compilation of rules for solving linear and quadratic equations, and has given rise to the word *algebra*. This stamp was issued by the Soviet Union in 1983 to commemorate the 1200th anniversary of his birth.

Muhammad ibn Mūsā al-Khwārizmī (c.780-850)

阿爾·花拉子米

*Al-Kitāb al-muhtasar fī hisab
al-jabr Wa-l-muqābalā*

[The condensed book on the calculation of
restoration and reduction] c.825

Algebra

= Latinized version
of *al-jabr*

al-jabr = restoration
(adding the same term
to both sides)

al-muqābala = reduction
(combining like terms)

$$\begin{array}{l} 3x + 2 \\ = 6 - 2x \end{array} \rightarrow \begin{array}{l} 5x + 2 \\ = 6 \end{array} \rightarrow \begin{array}{l} 5x \\ = 4 \end{array}$$

$$\begin{array}{l} 3x + 2 + 2x \\ = 6 - 2x + 2x \end{array}$$

1591 François Viète published
*In Arthem Analyticem
Isagogē (Introduction to the
Analytic Art)*

- logistica numerosa
- logistica speciosa

Quod est, Nullum non problema
solvere. (There is no problem
that cannot be solved!)

沒有問題是解決不了的！



François Viète
(1540-1603)

「朕自起身以來，每日同阿哥等察阿爾熱巴拉新法最難明白。他說比舊法易，看來比舊法難，錯處亦甚多，鶻突處也不少。…

還有言者：甲乘甲、乙乘乙，總無數目，即乘出來亦不知多少，看起來想是此人算法平平矣。」

康熙帝朱諭



康熙帝(愛新覺羅玄燁)

1654-1722



傅聖澤 (Jean-François Foucquet,
1665-1741), 《阿爾熱巴拉新法》
(1712)

“The old method uses numerical values, while the new method uses symbols that are **accommodating** (*tongrong jihaō* 通融記號).

Using this accommodating notation, it is easy to perform calculation, and it enables one to see the situation clearly so that one can focus on the method and understand the underlying rationale of the calculation. **The use of numerical value works only for a particular value, while the use of accommodating notation encompasses all values in general.**”

Jean-François Foucquet (傅聖澤)
《阿爾熱巴拉新法 (New Method of Aerrebala) 》 1712



設 x 為雞的數目，則。

這句話蘊含了一個重要的思想：
把某個或某些數值量看作為一般物體，對這些一般物體進行有如對數值量進行的運算。

在未解得方程之前，我們不知道它們取何數值。但至少我們心裏明白，由於它們代表某些數值，它們一定滿足某些基本法則，例如 $A \times B = B \times A$ 、
 $A \times (B + C) = A \times B + A \times C$ 等等。

只要我們能夠建立方程，便能夠有系統地利用這些基本法則把方程化簡以解答問題。

在不同文化中的數學古籍裏，都記載有不少涉及方程的問題，例如在中國古籍中的：

「持衣追客」	《九章算術》	卷六第十六題、
「二鼠穿垣」	《九章算術》	卷七第十二題、
「竹高折地」	《九章算術》	卷九第十二題、
「勾中容圓」	《九章算術》	卷九第十六題、
「雉兔同籠」	《孫子算經》	卷下第三十一題，

都可以建立方程並解之。

當時，還未建立起後來發展出來的解方程方法(如中國的天元術，或者西方的「阿爾熱巴拉新法」)，人們面對這類問題的時候，採用不同而巧妙的手法作推理計算，有時更借助圖解處理問題。這便有如今天在小學課堂碰到有些問題，不必動用方程也能解決一樣。

那麼，懂得一套解方程的方法有沒有優勢呢？要不要學習解方程的方法呢？

不曉得解方程的方法，面對這些問題的時候，並非無計可施，但需巧匠方成。

未必人人能成為巧匠，但曉得解方程的方法後，有望人人成為熟手工。

Solution of algebraic equations

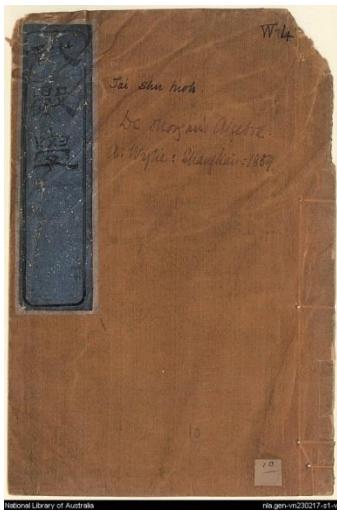
借根方算法 (calculation by borrowed root and powers) — 17th century

阿爾熱巴拉新法 (new method of *aerrebala*) — 18th century

天元術 (method of celestial unknown) — 12th/13th centuries,
revival of indigenous method in 18th century

代數術 (algebra) — mid 19th century

Augustus De Morgan, *Elements of Algebra*, 1835



代數學卷首

英國 樂摩甘譯

英國 儒烈亞力 口譯
海甯 李善蘭筆受

式准例作
又如

六甲甲乙丙乙丙丙

一一甲乙丙丙丙丙丙丙

一一甲乙丙丙丙丙丙丙

一一甲乙丙丙丙丙丙丙

凡代數諸分數並列，或分數與整數並列

代數學 卷首 緝領

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欲明代數，須先明數學，最要看分數小數之理，若未明，必先攷求之。此代數之捷徑也。

數學以本號寫數，號非必幾何，而以代幾何也。假如人有羊羣，以小石子數之，則石子為羊之號，算家所用之號，不用石子，而用筆畫，若一之號已定，則餘號俱定，假如言物之一段任若干長，或一尺，或一里，雖長短不同，俱命為一，則數學中或幾尺，或幾里，統謂之若干。「用」號於兩號之間，乃指兩號相加，如一為若干長，短一之號，則一為一之號，而一號變為二號更便也。由是一變為三，三一變為四，餘可類推。

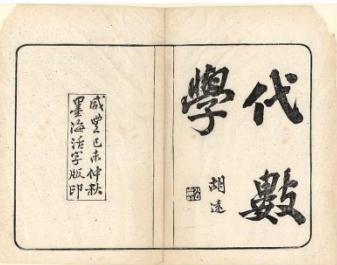
有物數有虛數，如一二三諸號，指一里二里三里等，或指一升二升三升等，則此諸號謂之物數，設除去物意，而空用一二三諸數，如謂二加三得五，則謂之代數學。

卷首 緝領

虛數學者習見，往往但明虛數，不知物與虛有兩種數間，數學中全用物數有若干術曰：止有二術，加與減也。如里與里可相加相減，若乘法則一二三諸數，不過言幾倍，如六里五乘之，是言五倍六里，即有兩種數，一為里是物數，一為倍是虛數，故乘法之法數必為倍數，若以物乘物，如六尺乘三尺，無是理也。設如一匹布，值二洋銀，問十二匹該若干，非以二洋銀乘十二匹布乎？不然，一匹值二洋銀，則每匹買者應出二洋銀，所以有十二倍二洋銀，則十二乃倍數，非匹數也。

約法，指倍物分物之意，即分全幾何為若干等分，如十八里路，以三里納之，即謂十八里中有幾倍三里，又或十八里路，以三納之，乃謂十八里分為三等分，每分有若干里也。十八里以三里約之，得六，即三里六倍之，成十八里。若十八里以三約之，得六里，即十八里分為三等分，每分六里也。若用虛數則兩術所同，即以三約十八，得六也。

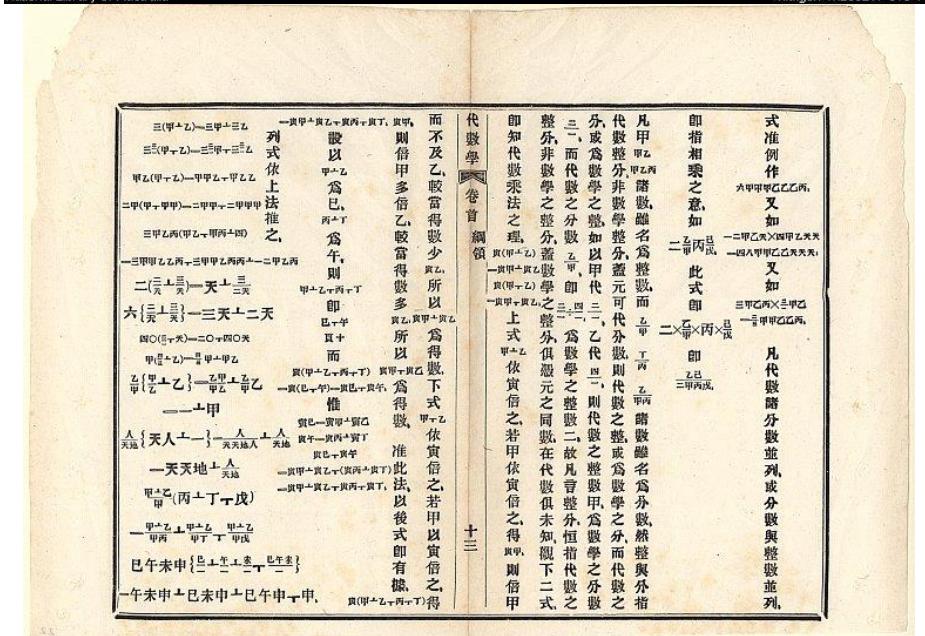
問十二尺乘幾倍八尺，答曰：多于一少于全，因一倍之若干分，心未明故也。言幾倍，乃如物跨行，非附行，其每跨為一倍，如一跨得八尺，而小于



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translated by
Alexander
Wylie and
Li Shanlan
(李善蘭)
in 1859



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The symbols of ARITHMETIC have a *determinate connexion*; for instance, 4 is always $2 + 2$ whatever the things mentioned may be, miles, feet, acres, &c. &c.

In ALGEBRA, we take symbols for numbers which have no determinate connexion.

數學之諸號，有一定相連屬之理。如四即 $\frac{二十二}{二十二}$ 之數，任何物，或里，或尺，或畝等，皆同至代數諸號，非一定相連屬。如數學之一二三諸號，或爲一尺二尺，或爲一里二里等，數皆一定，而代數諸號，言公數之理，所得非一定數。代數入門，須明此理，上所論約言其理，恐不能啟蒙，故復取數中公理之一明之。

I. **ALGEBRA** is the European corruption of an Arabic phrase, which may be thus written, *al jebre al mokabalah*, meaning *restoration and reduction*. [...]

II. A letter denotes a number, which may be, according to circumstances, as will hereafter appear, either any number we please; or some particular number which is not known, and which, therefore, has a sign to represent it till it is known; [...]

代數學，西名阿爾熱巴拉，乃亞喇伯言，譯卽補足相消也。今所存最古本，六朝時希臘丟番都所譯，或創造，或傳自東方諸國，未考。至唐時，便麼西傳此學于回回人，或云得之印度。印度所存最古本，名微斜迦尼大，南宋紹興時譯。寧宗時，便麼西之書，遞授至以大利薄那洗有譯本。明嘉靖三十六年，英醫生立可始傳其學。時尙未全以字代數，弟與明季所譯借根方同。
用字代數，或不定數，或未知之定數，俱以字代之。恒用之已知數，或因太繁，亦以字代，如周字代爲圓周率，又訥字代爲訥白爾對數底率。

「用字代數、或不定
數、或未知之定數，
俱以字代之。」

偉烈亞力、李善蘭合譯《代數學》(1859)
[August De Morgan, *Elements of Algebra*
(1835)]

「代數之法，無論何
數，皆可任以何記號
代之。」

傅蘭雅、華蘅芳合譯《代數術》(1872)
[William Wallace, *Algebra*, written for
Encyclopædia Britannica between 1801
and 1810]

代何數，二邊恒相等，如

$$\frac{\text{三甲} + \text{二甲}}{\text{甲}} = \text{甲}$$

此式甲字任代何數，兩邊之數必等，列自明之

式如下。

$$\text{甲} + \text{乙} = \text{乙} + \text{甲}$$

$$\text{甲} + \text{甲} = \text{二甲}$$

$$\frac{1}{2}\text{甲} + \frac{1}{2}\text{甲} = \text{甲}$$

$$\text{甲} + \text{一甲} = \text{甲} + \text{三甲}$$

$$\text{甲} + \text{甲} + \text{甲} = \text{三甲}$$

$$\frac{2}{3}\text{甲} + \frac{2}{3}\text{甲} + \frac{2}{3}\text{甲} = \text{甲}$$

$$\text{二甲} + \text{三甲} = \text{甲} + \text{四甲}$$

$$-\text{八甲} + \text{六甲} + \text{五甲} = -\text{甲}$$

又列二式，非能自明，而與理無不合。

$$\frac{-\text{甲} + \text{一甲} + \text{二甲} + \text{三甲} + \text{二甲} + \text{三甲}}{= \text{甲}} = \frac{-\text{甲} + \text{二甲} + \text{一甲}}{= \text{甲}}$$

偶方程式，偶合偶不合，故名偶式，如下。

$$\begin{matrix} \text{乙} + \text{一} \\ \text{甲} + \text{三} \end{matrix} = \begin{matrix} \text{七} \\ \text{二} \end{matrix}$$

此二式，若乙非代六，甲非代十

五，即不合，又此式，甲必與乙丙和數等，方合，凡有數，代式中之元字而合，

則此數爲足數，如則十五爲甲之足數，

Every collection of algebraical symbols is called an *expression*, and when two expressions are connected by the sign =, the whole is called an *equation*.

Arithmetical problem* and *Algebraical problem

數學之題，有某數，有某法，問以法推數得若干，如二十五與三百，兩數相乘爲實，如五十而一得若干。

代數學
卷首 紅領

三十一

- 1. Is there any such number? 2. If there be, by what operations [...] may it be found? 3. What is the result of these operations, or the number required.**

「一平方正，幾根負，
幾真數正。取根數二
分之一自乘，與真數
比。真數少或相等者，
有；真數多者，無。」

汪萊 · 《衡齋算學》 卷七

$$x^2 - bx + c = 0$$

若 $(b/2)^2 \geq c$ ，則有(實)根。

若 $(b/2)^2 < c$ ，則無(實)根。



《衡齋算學》(c. 1800)

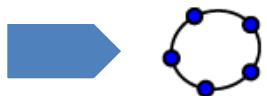


汪萊 (1768-1813)

開方術 Method of extracting square root



<http://ggbtu.be/m2744339>



《九章算術》

(成書於公元前一世紀至公元一世紀之間)

第四章：少廣・問題十二

「今有積五萬五千二百二十五步。

問：為方幾何？

答曰：二百三十五步。

開方。術曰：置積為實。借一籌，步之，超一等。議所得，以一乘所借一籌為法，而以除。除已，倍法為定法。……」

a^2		
	ab	\overline{b} + c
ab	b^2	
$c(a + b)$		c^2

$$(a + b + c)^2 = 55225$$

$$a \in \{0, 100, 200, \dots, 900\}$$

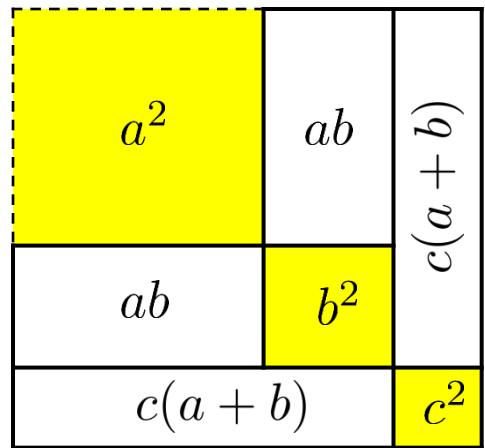
$$b \in \{0, 10, 20, \dots, 90\}$$

$$c \in \{0, 1, 2, \dots, 9\}$$

$$a = 200$$

$$a^2 = 40000$$

$$55225 - 40000 = 15225$$



$$b = 30$$

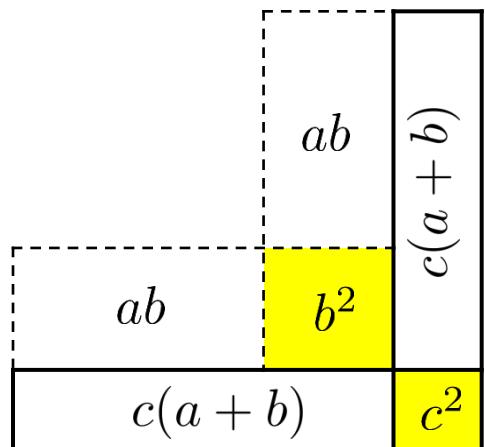
$$b^2 + 2ab = 12900$$

$$15225 - 12900 = 2325$$

$$c = 5$$

$$c^2 + 2c(a+b) = 2325$$

$$2325 - 2325 = 0$$



$$x = 200 + 30 + 5 = 235$$

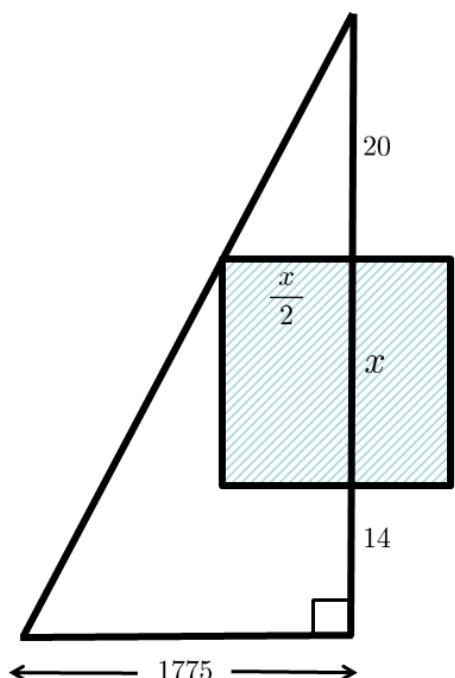
$$\begin{array}{r} \begin{array}{c} 2 \\ 5 \\ \hline 4 \\ 3 \\ \hline 4 \\ 6 \\ 5 \end{array} & \begin{array}{c} 3 \\ 52 \\ \hline 1 \\ 52 \\ \hline 1 \\ 29 \\ \hline 23 \\ 25 \\ \hline 23 \\ 25 \end{array} & \begin{array}{c} 5 \\ 25 \\ \hline 25 \\ \hline 25 \end{array} \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccccccccc}
 & 2 & 3 & 5 & \cdot & 0 & 3 \\
 & \underline{-} & \underline{-} & \underline{-} & & \underline{-} & \underline{-} \\
 5 & 52 & 40 & & \cdot & 00 & 00 & \dots
 \end{array} \\
 \begin{array}{c}
 2 \\
 4 \\
 \hline
 1 & 52
 \end{array} \\
 \begin{array}{c}
 4 \ 3 \\
 1 & 29 \\
 \hline
 23 & 40
 \end{array} \\
 \begin{array}{c}
 4 \ 6 \ 5 \\
 23 & 25 \\
 \hline
 15 & 00
 \end{array} \\
 \begin{array}{c}
 4 \ 7 \ 0 \ 0 \\
 00 & 00 \\
 \hline
 15 & 00 \ 00
 \end{array} \\
 \begin{array}{c}
 4 \ 7 \ 0 \ 0 \ 3 \\
 14 \\
 \hline
 89 & 91
 \end{array} \\
 \dots
 \end{array}$$

《九章算術》第九章第二十題

今有邑方不知大小，各開中門。
出北門二十步有木。出南門一
十四步，折而西行一千七百七
十五步見木。問：邑方幾何？

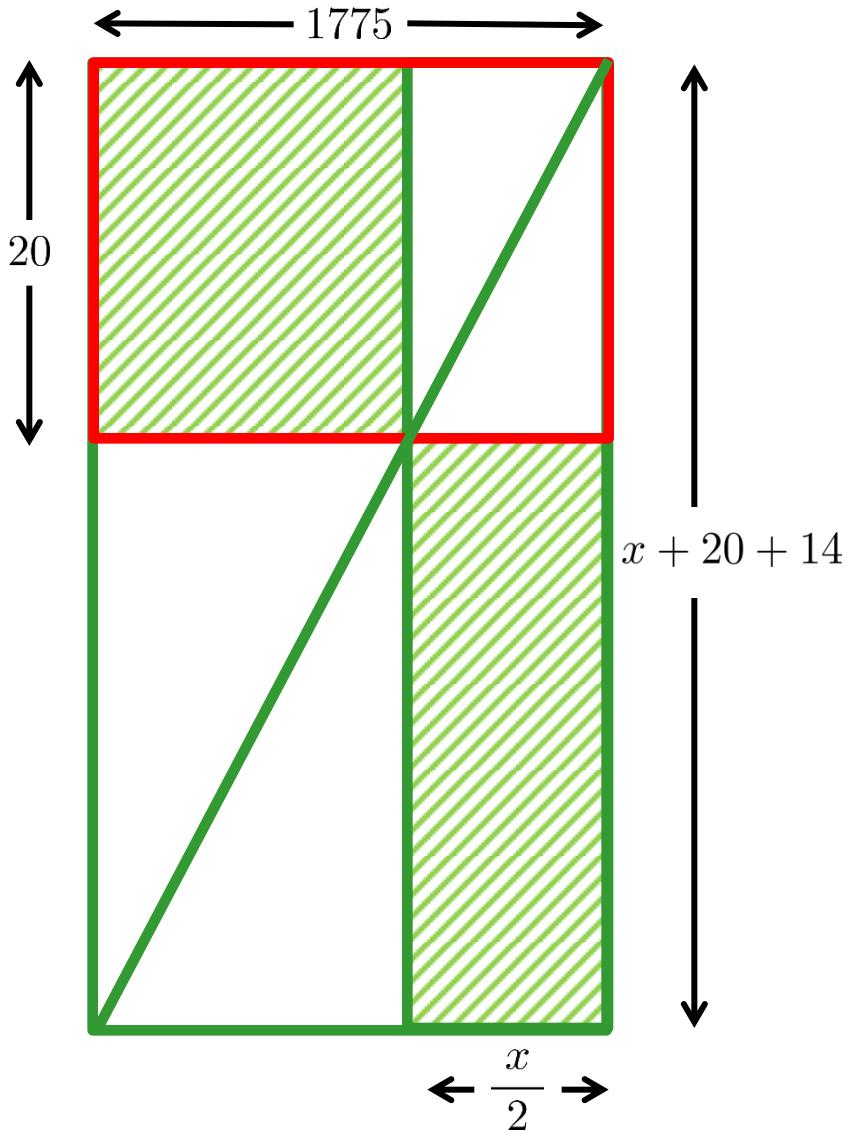
以今天數學語言表為一條二次方程求解



$$x^2 + 34x = 71000$$

$$\begin{aligned} x(x + 34) &= 2 \times 20 \times 1775 \\ &= 71000 \end{aligned}$$

帶從開方法



$$\frac{x}{2}(x + 20 + 14) = 20 \times 1775$$

$$x(x + 34) = 71000$$

以今天數學語言表為一條**二次方程求解**

$$x^2 + 34x - 71000 = 0.$$

$34a$	a^2	ab	$c(a + b)$
$34b$	ab	b^2	
$34c$	$c(a + b)$		c^2

$$(a + b + c)^2 + 34(a + b + c) = 71000$$

$$a \in \{0, 100, 200, \dots, 900\}$$

$$b \in \{0, 10, 20, \dots, 90\}$$

$$c \in \{0, 1, 2, \dots, 9\}$$

$$a = 200 \quad a^2 + 34a = 46800$$

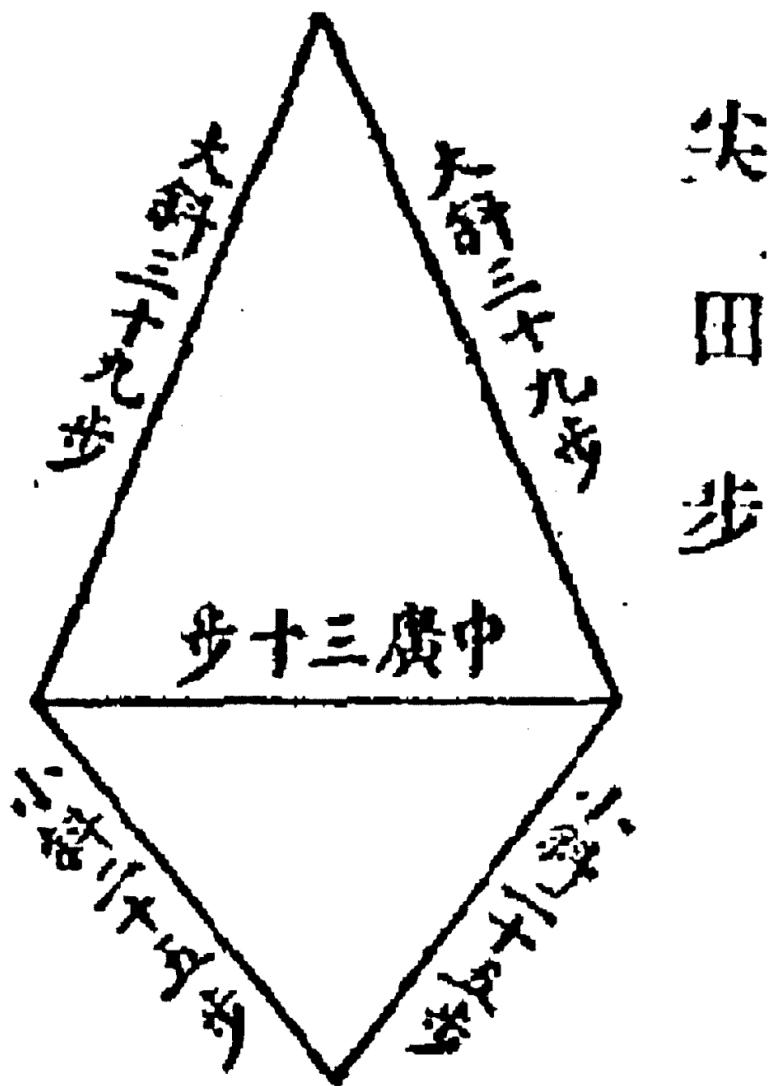
$$71000 - 46800 = 24200$$

$$b = 50 \quad b^2 + 2ab + 34b = 24200$$

$$24200 - 24200 = 0$$

$$c = 0$$

$$x = 200 + 50 + 0 = 250$$



《數書九章》卷五題一：

問有兩尖田一段，其尖長不等，兩大斜三十九步，兩小斜二十五步，中廣三十步，欲知其積幾何？

III

正負開三乘方圖

術曰商常爲正

實常爲負

從常爲正

益常爲負

III

商運位
益運位

III

商實方

數書九章卷五

正除堂數書

III

○正

運位
益運位
益而數再
益而數再

III

商

III

商

III

商

III

商

益下廉
益而生兩
益八百爲

$$P(Y) = -Y^4 + 763200 Y^2 - 40642560000 = 0.$$

$$\begin{aligned} P(800) &= -800^4 + 763200 \times 800^2 - 40642560000 \\ &= 38205440000 > 0. \end{aligned}$$

Set $Y = X + 800$,

$$\begin{aligned} P(X + 800) &= -(X + 800)^4 + 763200 (X + 800)^2 - 40642560000 \\ &= -X^4 - 3200 X^3 - 3076800 X^2 \\ &\quad - 826880000 X + 38205440000 \end{aligned}$$

Solve $P(X + 800) = 0$ and obtain a solution $X = 40$.

Hence, $Y = 40 + 800 = 840$ is a solution of the original equation $P(Y) = 0$.

$$P(X) = 2X^3 - 10X^2 - 5X + 7$$

$$P(X + 3) = ? \quad P(3) = ?$$

$$\begin{array}{r} 2 \quad -10 \quad -5 \quad 7 \\ 0 \quad 6 \quad -12 \quad -51 \\ \hline 2 \quad -4 \quad -17 \quad -44 \\ 0 \quad 6 \quad 6 \\ \hline 2 \quad 2 \quad -11 \\ 0 \quad 6 \\ \hline 2 \quad 8 \\ 0 \\ \hline 2 \end{array}$$

$$P(X + 3) = 2X^3 + 8X^2 - 11X - 44$$

$$P(3) = -44$$

$$P(X) = a_3 X^3 + a_2 X^2 + a_1 X + a_0$$

An algorithm to compute
 $P(X + \alpha)$ and $P(\alpha)$

$$\begin{array}{cccc} a_3 & a_2 & a_1 & a_0 \\ 0 & b_3 \alpha & b_2 \alpha & b_1 \alpha \\ \hline -\hline b_3 & b_2 & b_1 & b_0 \end{array}$$

where $b_3 = a_3$,
 $b_2 = a_2 + b_3 \alpha$,
 $b_1 = a_1 + b_2 \alpha$,
 $b_0 = a_0 + b_1 \alpha$.

Repeat the procedure to the bottom row. Stop when a single term is obtained.

a_3	a_2	a_1	a_0
b_3	b_2	b_1	b_0
c_3	c_2	c_1	
d_3	d_2		
e_3			

$$P(X + \alpha) = e_3 X^3 + d_2 X^2 + c_1 X + b_0$$

$$P(\alpha) = b_0$$

The same procedure
works in the general case
of a polynomial of degree N .

the known arithmetical process for extracting the square root.

g. At Cubic equations, the aberration of the old practice of evolution commences; and our theorem places us at once on new ground. We have here

$$\Delta = ax + bx^2 + x^3$$

and must proceed thus:

1	b	a	\$\Delta(r+r+\dots)\$
$\frac{r}{B}$		$\frac{Br=0}{A}$	$\frac{-Ar}{\Delta}$
$\frac{r^2}{B^2}$		$\frac{B+r^2=B}{A}$	$\frac{-A'r^2}{\Delta}$
$\frac{r^3}{B^3}$		$\frac{B'+r^3=B}{A'}$	$\frac{\&c.}{\Delta'}$
&c.			

This ought to be the arithmetical practice of the cube root, as an example will prove.

Ex. I. Extract the cube root of 48228544.

Having distributed the number into tridigital periods as usual, we immediately perceive that the first figure of the root is 3 = R. Consequently, the first subtrahend is $R^3=27$, the first derivee $gR^2=27$, the second $gR=9$; the third (=1,) need not be written. Hence

			48228544(364
9.	27..	27	
6	576	—	21228
—	—	—	19656
96	3276	—	—
12.	612..	—	—
4	4336	—	1572544
—	393136	—	1572544
1084			

In this example the reader will perceive that no supplementary operations are concealed. The work before him is complete, and may be verified mentally. I need not intimate

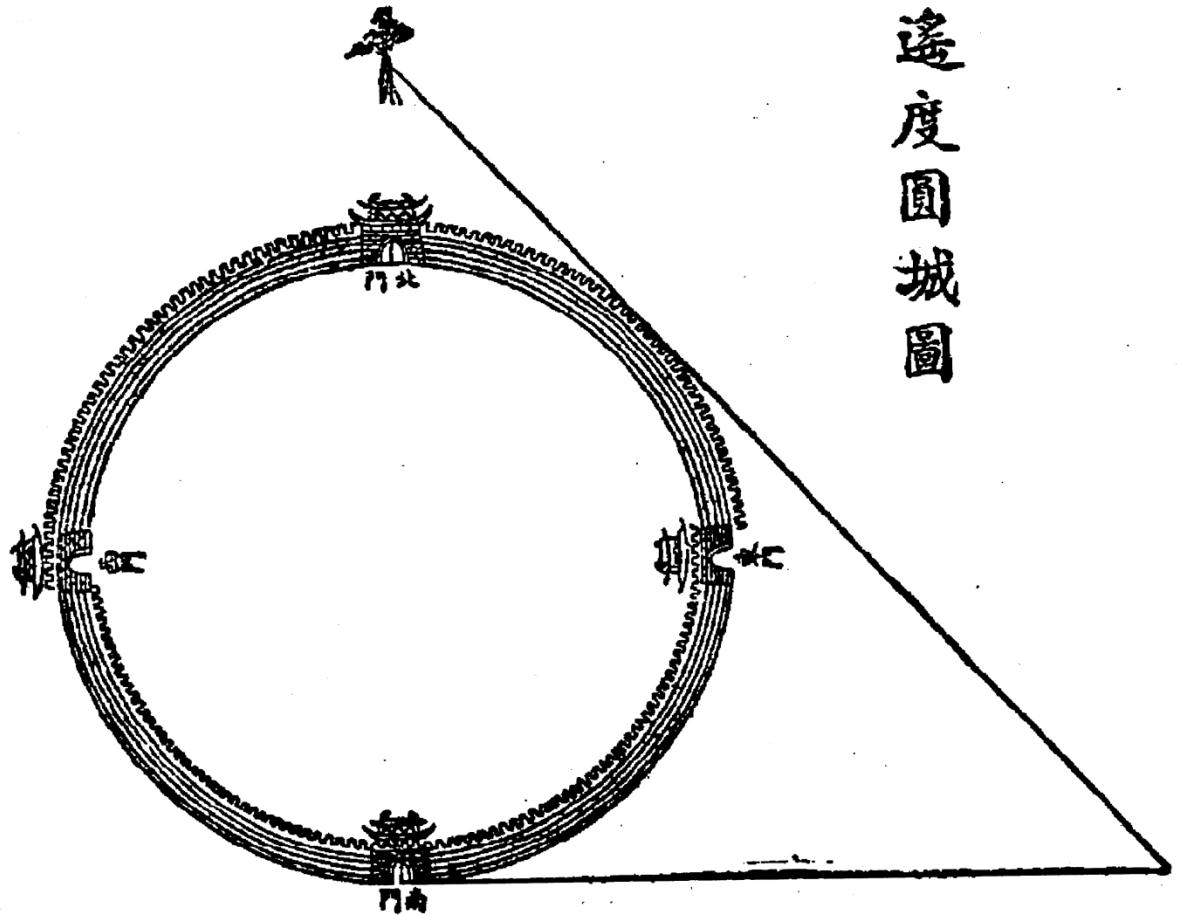
W.G. Horner, A new method of solving numerical equations of all orders by continuous approximation, *Philosophical Transactions of the Royal Society of London*, 109 (1819), 308-335.

“The elementary character of the subject was the professed objection; his recondite mode of treating it was the professed passport for its admission.”

(T.S. Davis) !

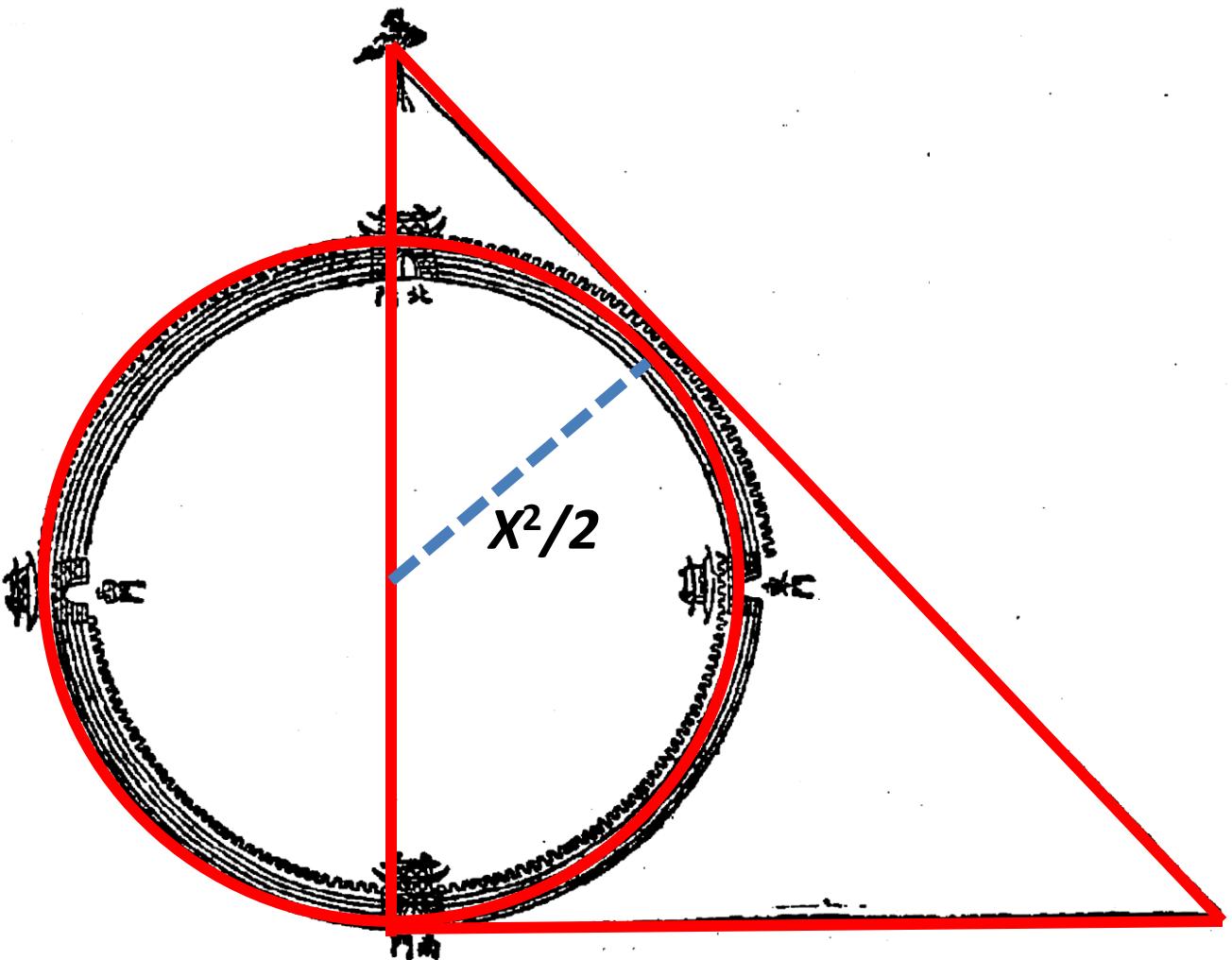
Same principle as the method given by Qin Jiu-shao (秦九韶) in 《數書九章 [Mathematical Treatise in Nine Sections]》 (1247)

遙度圓城圖



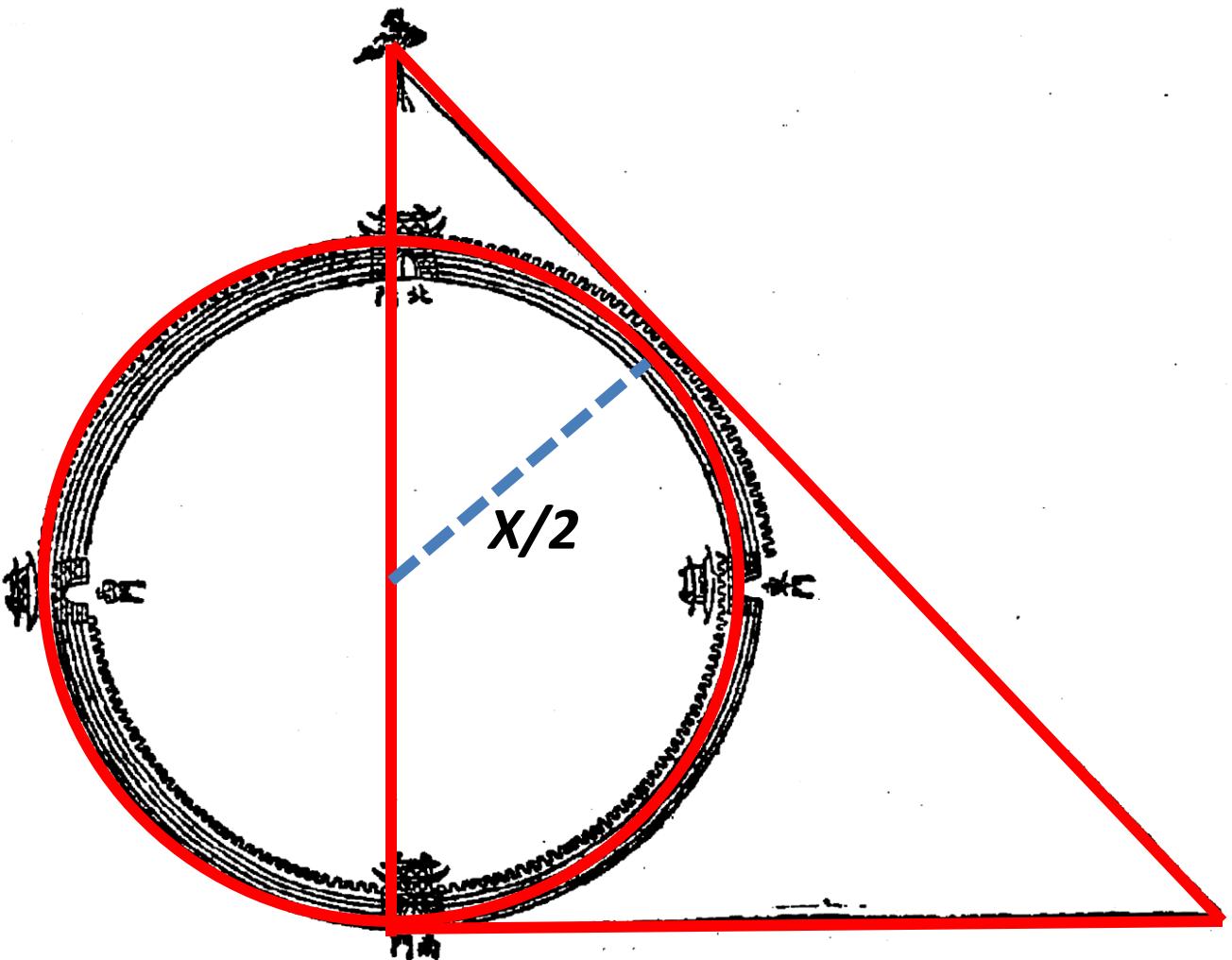
《數書九章》卷八題二：

問有圓城不知周、徑，四門中開。北外三里有喬木，出南門便折東行九里，乃見木。欲知城周、徑各幾何？圓用古法。



Let X^2 be the diameter.
Solve the equation

$$P(X) = X^{10} + 15X^8 + 72X^6 - 864X^4 - 11664X^2 - 34992 = 0.$$



Let X be the diameter.
Solve the equation

$$P(X) = X^3 + 3X^2 - 972 = 0.$$

A similar problem was treated by Li Ye [李冶 1192-1279]
as Problem 4 in Book 3 of his book 《測圓海鏡》
[Sea Mirror of Circle Measurement] of 1248,
resulting in solving a cubic equation.

四庫館於《數書九章》
卷八題二有如下按語
[李銳 (1768-1817)]：

「此題非甚難者，乃
取至九乘方，未得
其要，徒多曲折耳。」



李銳 (1768-1817)

《數書九章》卷八題二

川益上廉

川北里

川三加生川實

以上乘副得次

已上係求率圖

以後係開方圖

○商正

-34992

川三加生川實貞星

○方虛

-11664

川一丁上川上廉

次虛

-864

川上川才廉貞

雜虛

72

土正行廉

爻虛

15

星正下廉

○下虛

1

約實商三里。

二四

3

1

以商生隅得下
廉以商生下廉

得星廉

川商

川三加生川威

川三加生川威

○

川一丁上川

○

川上川

○

土正

○

24
星廉

爻廉

以商生星廉得
下廉

下廉

隅

-	34992	34992	0
0	11664	11664	
- 11664	15552	3888	
0	5184	5184	
- 864	2592	1728	
0	864	864	
72	216	288	
0	72	72	
15	add -----> 9	24	Multiply by 3
0	add -----> 3	3	Multiply by 3
1	add -----> 0	1	Multiply by 3

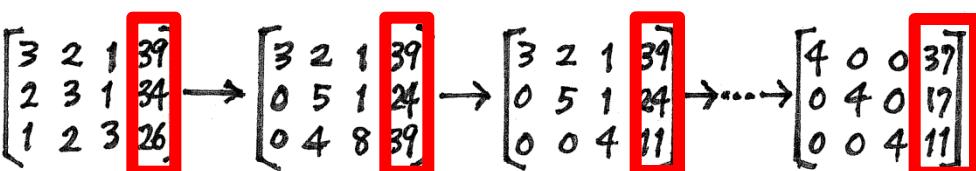
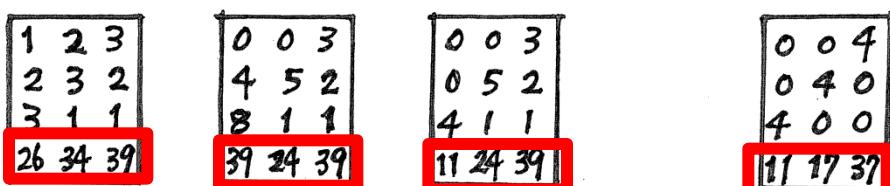
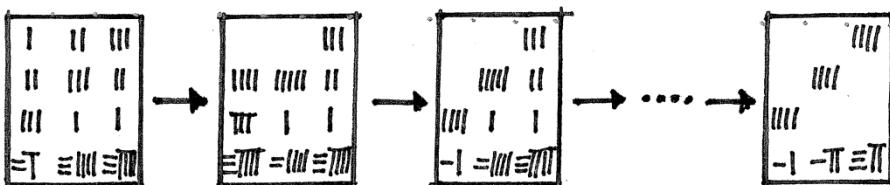
$P(X) = X^{10} + 15X^8 + 72X^6 - 864X^4 - 11664X^2 - 34992$,
 $P(3) = 0$. Hence, 3 is a solution to the equation $P(X) = 0$.

《九章算術》第八章 「方程」第一題

今有上禾三秉，中禾二秉，下禾一秉，實三十九斗；上禾二秉，中禾三秉，下禾一秉，實三十四斗；上禾一秉，中禾二秉，下禾三秉，實二十六斗。問：上、中、下禾實一秉各幾何？

現代數學表示式

$$\begin{aligned}3x + 2y + z &= 39 \\2x + 3y + z &= 34 \\x + 2y + 3z &= 26\end{aligned}$$



Gauss 消去法 (十九世紀初)

線性方程組

《九章算術》第八章
公元前一世紀至公元一世紀之間

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= C_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= C_2 \\ \cdots\cdots \\ a_{k1}x_1 + a_{k2}x_2 + \dots + a_{kn}x_n &= C_k \end{aligned}$$

線性同餘方程組

《孫子算經》, 公元四五世紀
《數書九章》, 1247.

$$\begin{aligned} x &\equiv A_1 \pmod{m_1} \\ x &\equiv A_2 \pmod{m_2} \\ \cdots\cdots \\ x &\equiv A_k \pmod{m_k} \end{aligned}$$

70
21
15

3
5
7

$$2 \times 70 = 140$$
$$3 \times 21 = 63$$
$$2 \times 15 = 30$$

相加得 233.
以數 105 的倍
作修訂得
128, 23, 338,
443, 等等

○物不知總

六〇

孫子歌曰

又云韓信點兵也

三人同行七十稀

五樹梅花廿一枝

七子團圓正半月

除百令五便得知

今有物不知數只云三數剩二箇五數剩三箇七數剩二

箇問共若干

荅曰 共二十三箇

答案是
23

法曰列三五七維乘以三乘五得一十又以七乘之得

一百零五爲滿法數列位○另以三乘五得一十爲七數

剩一之衰○又以三乘七得二十爲五數剩一之衰

○又以五乘七得五十倍作七以三除之餘一故用七

程大位
《算法統宗》
(1592)

三人同行七十稀，
五樹梅花廿一枝。
七子團圓正半月，
除百令五便得知。

**Da Yan 大衍
(Great Extension)
Art of Searching for Unity**

The general principles of the *Ta-yen* are probably given in their simplest form, in the above rudimentary problem of Sun Tsze; Subsequent authors enlarging on the idea, applied it with much effect to that complex system of cycles and epicycles which form such a prominent feature in the middle-age astronomy of the Chinese. The reputed originator of this theory as applied to astronomy is the priest Yih Hing who had scarcely finished the rough draft of his work **大衍曆書**

Ta-yen leih shoo, when he died A.D. 717.

But it is in the "Nine sections of the art of numbers" by Tsin Keu chaou that we have the most full and explicit details on this subject. Here we have the various applications of this theory worked out at great length; the first problem being to find a solution of a passage in the Yih King treating of the origin of the divining numbers:—

Qu. In the Yih King it is said,—'The Great Extension number is 50, and the Use number is

* Native writers are divided in opinion as to the time when Sun Tsze lived; some consider him the same as Sun Woo-tze, a military officer during the Heptarchy about B.C. 220. The more probable opinion however, is that he lived towards the end of the Han or during the Wei dynasty in the third century of the Christian era.

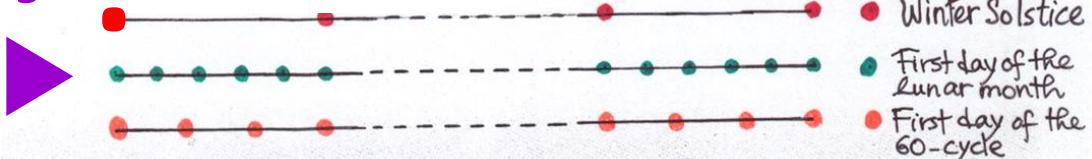
**秦九韶
(QIN Jiu-shao)
《數書九章》
1247
大衍求一術**

Shushu Jiuzhang 數書九章, Book I, Problem 2

古曆會積

Let the solar year be equal to $365\frac{1}{4}$ days, the moon's revolution, $29\frac{499}{940}$ days, and the Kea-tsze, 60 days. Suppose in the year A.D. 1246, the 53d day of the Kea-tsze or sexagenary cycle of days is the 1st of the 11th month; the 57th day of the Kea-tsze is the Winter solstice or 1st day of the solar year; and the 1st day of the Kea-tsze is the 9th day of the month. Required the time between two conjunctions of the commencement of these three cycles; also, the time that has already elapsed, and how much has yet to run. Ans. The time between two conjunctions, 18,240 years: 225,600 months: 6,662,160 days: number of years already past 9,163: number of years unexpired, 9,077.

Shang Yuan 上元



N days in a Shang Yuan period

$$N \equiv 0 \pmod{365\frac{1}{4}}$$

$$N \equiv 0 \pmod{29\frac{499}{940}}$$

$$N \equiv 0 \pmod{60}$$

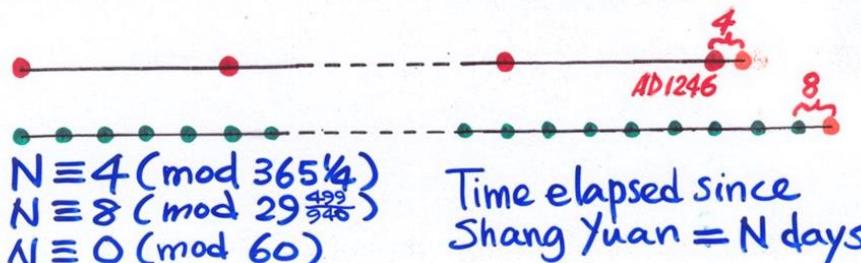
$$x = 4 \times 940 \times N$$

$$x \equiv 0 \pmod{1373340}$$

$$x \equiv 0 \pmod{111036}$$

$$x \equiv 0 \pmod{225600}$$

$$x = 2087476800, N=555180$$



about calendrical reckoning

THEOREM 17 (CHINESE REMAINDER THEOREM). * A Dedekind domain R possesses the following property:

(CRT) Given a finite number of ideals α_i and of elements x_i of R ($i = 1, \dots, n$), the system of congruences $x \equiv x_i \pmod{\alpha_i}$ admits a solution x in R if and only if these congruences are pairwise compatible, that is, if and only if we have $x_i \equiv x_j \pmod{\alpha_i + \alpha_j}$ for $i \neq j$.

PROOF. The property (CRT) is related to the fact that in the set of ideals of a Dedekind domain R , each of the operations \cap and $+$ is *distributive* with respect to the other; that is, that given three ideals α, β, β' in R , we have:

$$\alpha \cap (\beta + \beta') = (\alpha \cap \beta) + (\alpha \cap \beta')$$

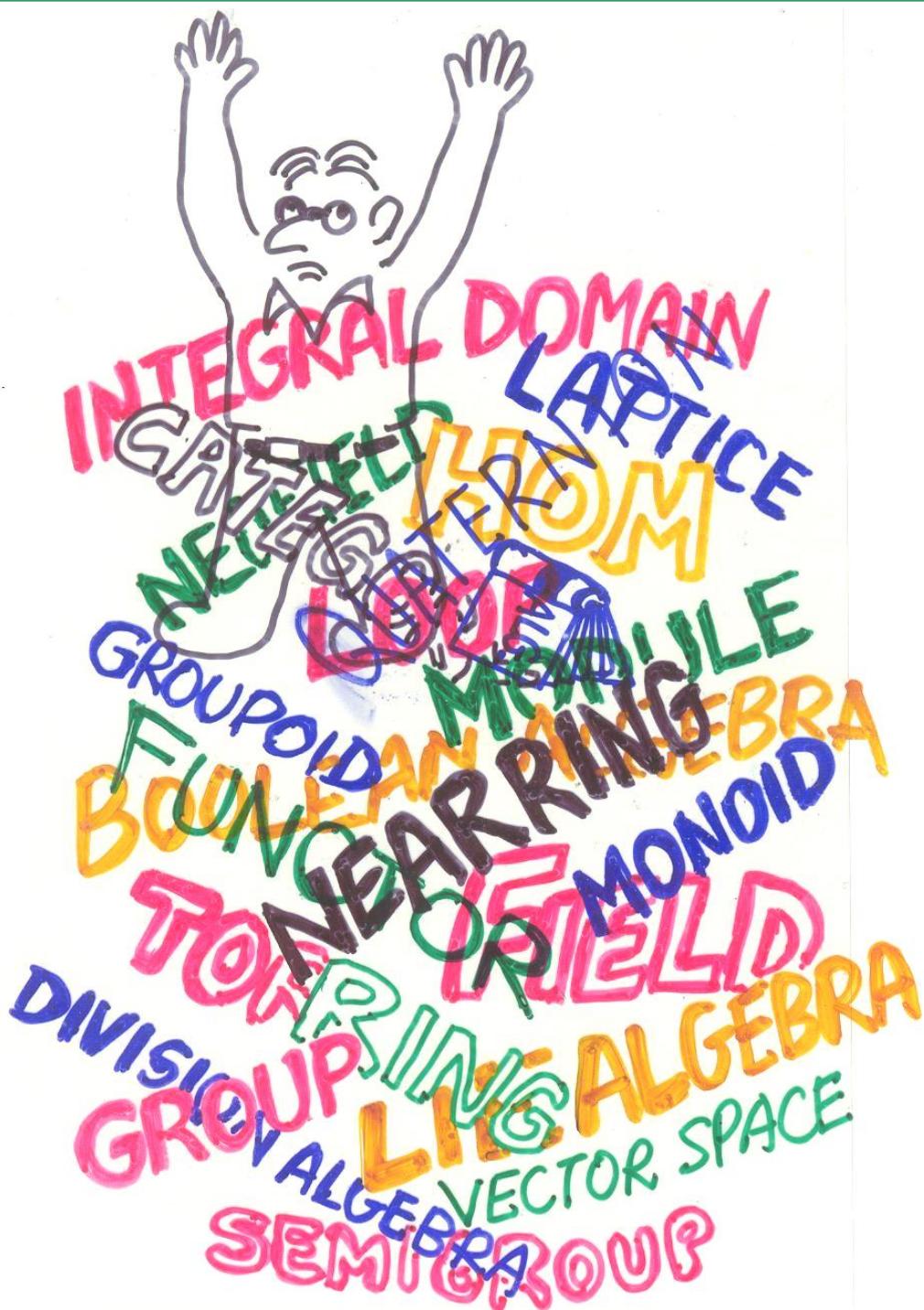
$$\alpha + (\beta \cap \beta') = (\alpha + \beta) \cap (\alpha + \beta').$$

* A rule for the solution of simultaneous linear congruences, essentially equivalent with Theorem 17 in the case of the ring J of integers, was found by Chinese calendar makers between the fourth and the seventh centuries A.D. It was used for finding the common periods to several cycles of astronomical phenomena.

Chinese Remainder Theorem

**Oscar Zariski & Pierre Samuel,
Commutative Algebra, Vol. I
(1958), Chapter V, p.279.**

What is (Abstract) Algebra?



MATH2301 is too difficult for me. I am going to flunk it.

Don't worry.
Abstract algebra is easy.

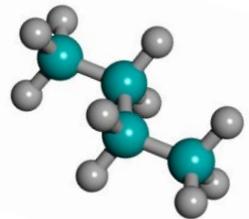
Why do I have to study all this abstract stuff? I do not see how it is used in my other courses nor in daily life. I do not see why it is called "algebra" either. It looks entirely different from the kind of algebra I learnt in school. I get confused with the multitude of definitions and theorems about things I cannot visualize.

All the notions are clearly defined and all the theorems are systematically derived. Everything is so precise and logical. You can even start from scratch without having to know much beforehand. How come you do not understand what is going on and feel confused?

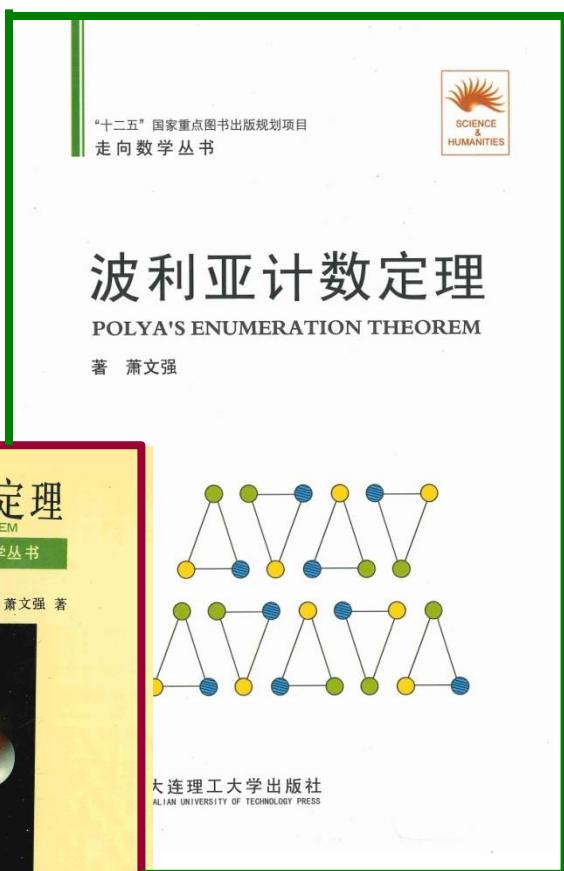


同分異構體 (isomer)

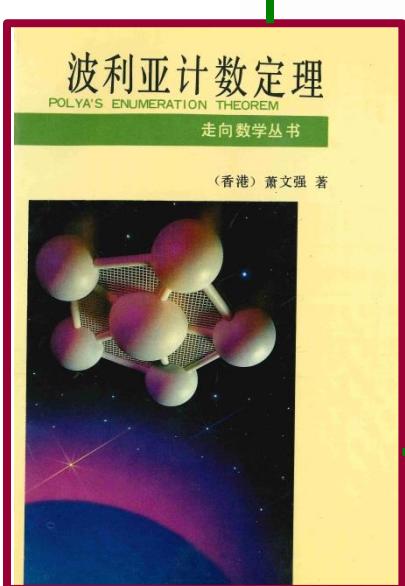
Butane 丁烷	C_4H_{10}	<pre> H H H H H - C - C - C - C - H H H H H </pre>	
Methylpropane 甲基丙烷	C_4H_{10}	<pre> H H H - C - H H H - C - C - C - H H H H </pre>	



共有多少個 C_4H_{10} 的同分異構體？

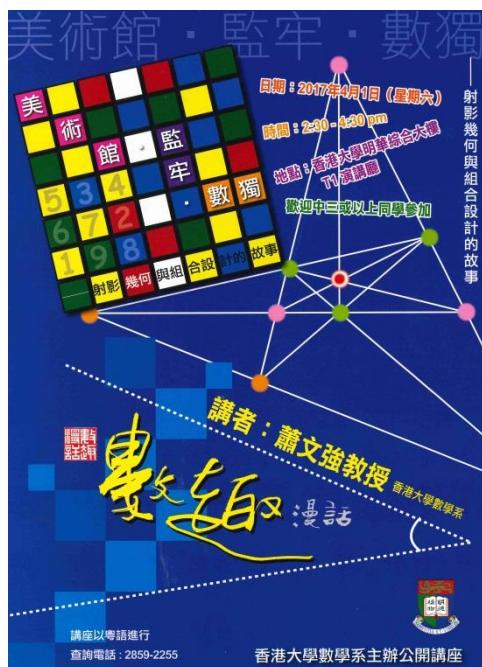
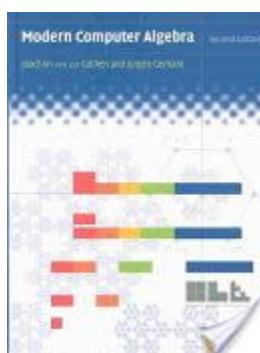
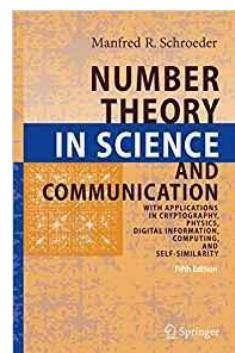
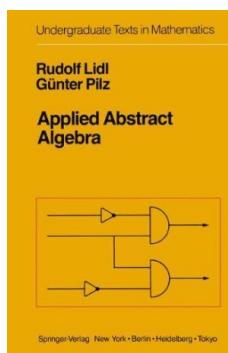
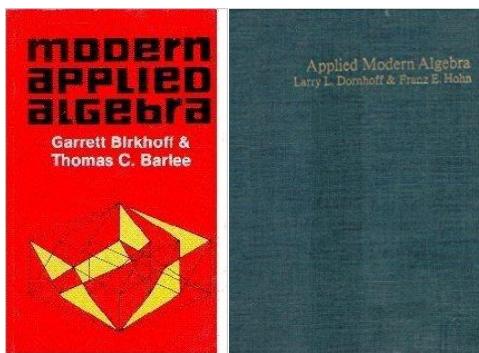


G. Pólya,
Kombinatorische
Anzahlbestimmungen
für Gruppen, Graphen
und Chemische
Verbindungen,
Acta Mathematica,
1937 (68), 145-254.



蕭文強，《波利亞計數定理》
1991年初版，2011年再版。

- ❖ G. Birkhoff, T. C. Bartee, *Modern Applied Algebra*, 1970.
- ❖ L. L. Dornhoff, F. E. Hohn, *Applied Modern Algebra*, 1978.
- ❖ R. Lidl, G. Pilz, *Applied Abstract Algebra*, 1984; 2nd edition, 1998.
- ❖ M. R. Schroeder, *Number Theory in Science and Communication*, 1984; 2nd edition, 1990.
- ❖ J. von zur Gathen, J. Gerhard, *Modern Computer Algebra*, 1999, 2nd edition, 2003; 3rd edition, 2013.



蕭文強, 數趣漫話：
美術館・監牢・數獨
—— 射影幾何與組合設計
的故事

2017年4月1日

EQUATION

$$3x = 4$$

$$2x^2 - 7x - 4 = 0$$

$$x^3 - 7x^2 + 9 = 0$$

.....

$$\sin x = x$$

$$\tan x + \tan 2x = 1$$

.....

$$x_0 = x_1 = 1$$

$$x_{n+2} = x_n + x_{n+1}, n \geq 0$$

.....

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = e^x$$

.....

FUNCTIONS

$$a_{11}x_1 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + \cdots + a_{2n}x_n = b_2$$

:

:

:

$$a_{m1}x_1 + \cdots + a_{mn}x_n = b_m$$

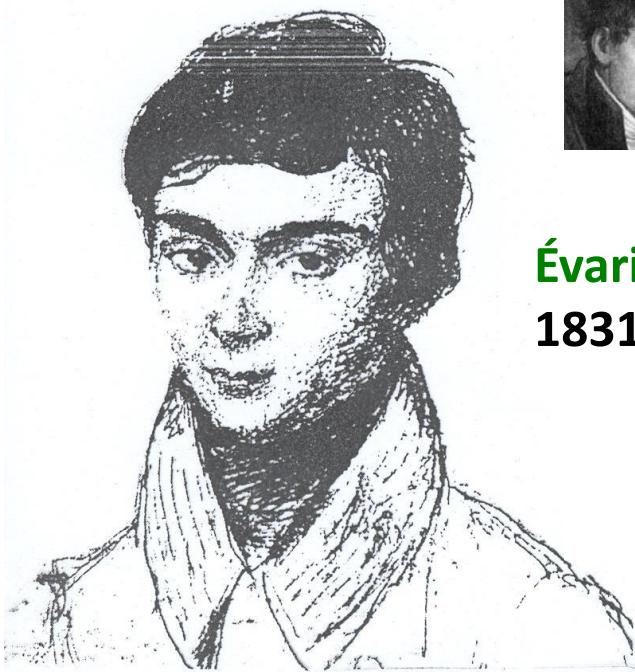


Is the quintic equation solvable by radicals?

Joseph Louis Lagrange (1736-1813)
1770/1771 *Réflexions sur la résolution*
algébrique des équations

Niels Henrik Abel (1802-1829)
1824 *Mémoire sur les équations*
algébriques

Paolo Ruffini (1765-1822)
1799 *Teoria Generale delle Equazioni, in cui si dimostra impossibile la soluzione algebraica delle equazioni generali di grado superiore al quarto*



Évariste Galois (1811-1832)
1831 *Mémoire sur les conditions*
de résolubilité des équations
par radicaux

Solving algebraic equations

$$ax^2 + bx + c = 0,$$

$$ax^3 + bx^2 + cx + d = 0,$$

.....

“Solvable by radicals”

Permutation of roots

Group of Permutations

Group

Galois Theory

$\mathbb{Q}, \mathbb{R}, \mathbb{C} \rightarrow$ Field

$\mathbb{Z} \rightarrow$ Ring

Group

Integral Domain

— UFD, PID,

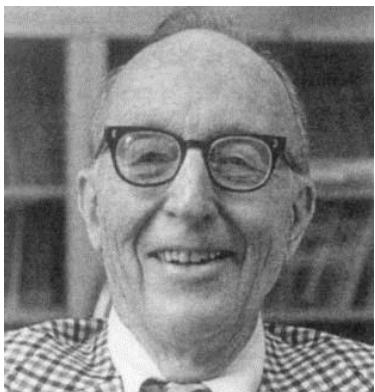
Euclidean Domain

Quotient Field
Extension Field
Algebraic Closure

- ❖ 蕭文強, 從方程到羣的故事, 《1, 2, 3, … 以外》, 第五章, 1990/1993/1994.
- ❖ M.K. Siu, On the learning and teaching of tertiary algebra [unpublished expanded version of “Why is it difficult to teach abstract algebra?”, in *Proceedings of the 12th ICMI Study Conference: The Future of the Teaching and Learning of Algebra*, 2001, 541-547, published in Italian in *Progetto Alice*, 10 (29), 2009, 311-330.]

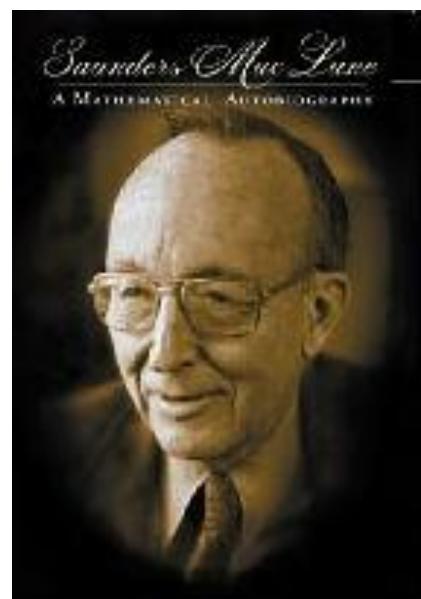
What is abstract algebra?

“... the program of studying algebraic manipulations on arbitrary objects with the intent of obtaining theorems and results deep enough to give substantial information about the prior existing particular objects.”



Saunders MacLane,
(1909-2005).

Saunders MacLane,
History of abstract
algebra: Origin, rise
and decline of a
movement, *Texas Tech.
Univ. Math. Series*, 13
(1981), 3-35.



以前見山是山，
見水是水。
後來見山不是山，
見水不是水。
如今見山又是山，
見水又是水。

青原惟信
(宋)《五燈會元》卷十七

謝謝香港數理教育學會的邀請，讓我有此機會與大家談談數學。

謝謝梁子傑老師應允作回應嘉賓，與大家分享他的高明識見。

柯志明先生協助製作 *GeoGebra* 顯示，以輔助講解，香港大學數學系呂美美女士協助製作圖片，為講座添色，謹此一併致謝。